EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. Find the equation of a circle which touches the line $x + y = 5$ at the point $(-2, 7)$ and cuts the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ orthogonally

2. Given that a right angled trapezium has an inscribed circle. Prove that the length of the right angled leg is the Harmonic mean of the lengths of bases.

3. A variable circle passes through the point A(a, b) & touches the x-axis; show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$.

4. Find the equation of the circle passing through the point $(-6, 0)$ if the power of the point $(1, 1)$ w.r.t. the circle is 5 and it cuts the circle $x^2+y^2-4x - 6y-3=0$ orthogonally.

5. Consider a family of circles passing through two fixed points A(3, 7) & B(6, 5). The chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinates of this point.

6. Find the equation of circle passing through (1, 1) belonging to the system of co-axial circles that are tangent at (2, 2) to the locus of the point of intersection of mutually perpendicular tangent to the circle $x^2 + y^2 = 4$.

7. Find the locus of the mid point of all chords of the circle $x^2 + y^2 - 2x - 2y = 0$ such that the pair of lines joining (0, 0) & the point of intersection of the chords with the circles make equal angle with axis of x.

8. The circle C : $x^2 + y^2 + kx + (1 + k)y - (k + 1) = 0$ passes through two fixed points for every real number k. Find.
(i) the coordinates of these two points.
(ii) the minimum value of the radius of a circle C.

9. Find the equation of a circle which is co-axial with circles $2x^2 + 2y^2 - 2x + 6y - 3 = 0$ &
$x^2 + y^2 + 4x + 2y + 1 = 0$. It is given that the centre of the circle to be determined lies on the radical axis of these two circles.

10. The circles, which cut the family of circles passing through the fixed points $A \equiv (2, 1)$ and $B \equiv (4, 3)$ orthogonally, pass through two fixed points (x_1, y_1) and (x_2, y_2), which may be real or imaginary. Find the value of $\left(x_1^3 + x_2^3 + y_1^3 + y_2^3\right)$.

11. A circle with centre in the first quadrant is tangent to $y = x + 10$, $y = x - 6$, and the y-axis. Let (h, k) be the centre of the circle. If the value of $(h + k) = a + b\sqrt{a}$ where $\sqrt{a}$ is a surd, find the value of a + b.

12. A circle C is tangent to the x and y axis in the first quadrant at the points P and Q respectively. BC and AD are parallel tangents to the circle with slope -1. If the points A and B are on the y-axis while C and D are on the x-axis and the area of the figure ABCD is $900\sqrt{2}$ sq. units then find the radius of the circle.

13. Let A, B, C be real numbers such that
(i) (sin A, cos B) lies on a unit circle centred at origin.
(ii) tan C and cot C are defined.
If the minimum value of $(\tan C - \sin A)^2 + (\cot C - \cos B)^2$ is $a + b\sqrt{2}$ where $a, b \in I$, find the value of $a^3 + b^3$.

14. An isosceles right angled triangle whose sides are 1, 1, $\sqrt{2}$ lies entirely in the first quadrant with the ends of the hypotenuse on the coordinate axes. If it slides prove that the locus of its centroid is $(3x - y)^2 + (x - 3y)^2 = \frac{32}{9}$.

15. A rhombus ABCD has sides of length 10. A circle with centre 'A' passes through C (the opposite vertex) likewise, a circle with centre B passes through D. If the two circles are tangent to each other. Find the area of the rhombus.

16. Find the equation of a circle which touches the lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$ and is contained in the given circle.

17. Find the equation of the circle which passes through the origin, meets the x-axis orthogonally & cuts the circle $x^2 + y^2 = a^2$ at an angle of 45º.

18. Circles C_1 and C_2 are externally tangent and they are both internally tangent to the circle C_3. The radii of C_1 and C_2 are 4 and 10, respectively and the centres of the three circles are collinear. A chord of C_3 is also a common internal tangent of C_1 and C_2. Given that the length of the chord is $\frac{m\sqrt{n}}{p}$ where m, n and p are positive integers, m and p are relatively prime and n is not divisible by the square of any prime, find the value of (m + n + p).